Ishlinsky Institute for Problems in Mechanics
of the Russian Academy of Sciences (IPMech RAS)

Pr. Vernadskogo, 101-1 Moscow 119526 Russia Phone: +7 495 434 0017 Fax: +7 499 739 9531 E-mail: ipm@ipmnet.ru

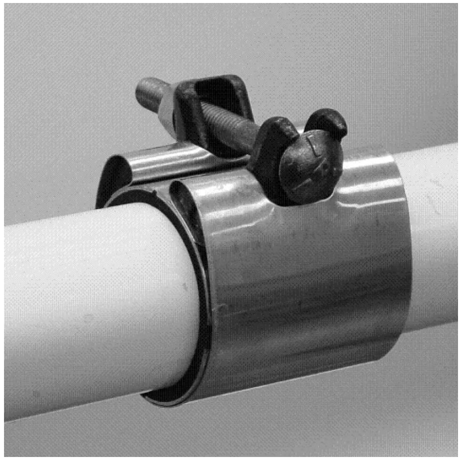
Kazakov K.E.

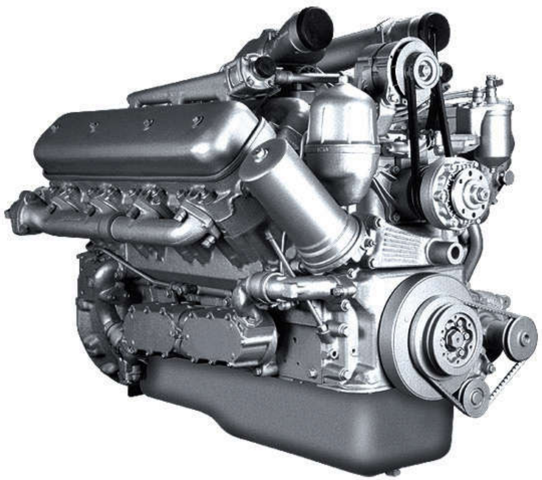
On accounting complex forms of surfaces in the task of interaction of a rigid insert and pipe with internal coating

15th International Conference on
Mechanics, Resource and Diagnostics of Materials and Structures
December 20-24, 2021; Ekaterinburg, Russia

Contact mechanics and its applications

- wheel-rail contact
- calculation of couplings
- brakes
- tires
- bearings
- internal combustion engines
- etc.



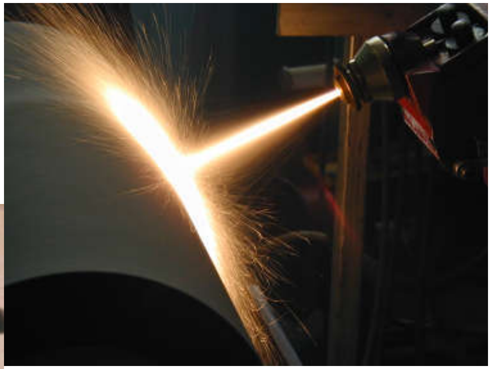


Protection coatings

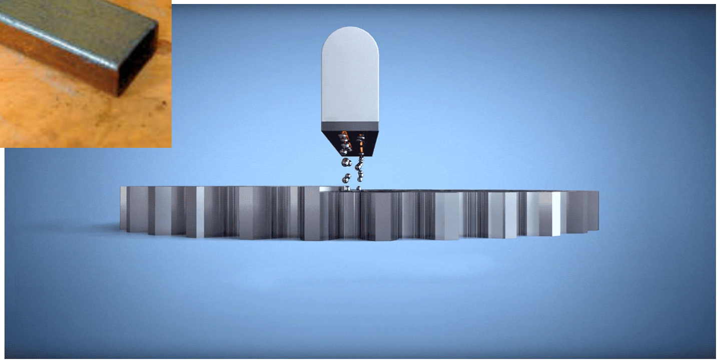
- thermal or electrical insulation
- protection against aggressive environments
- safety requirements
- leveling layer
- etc.

Coating

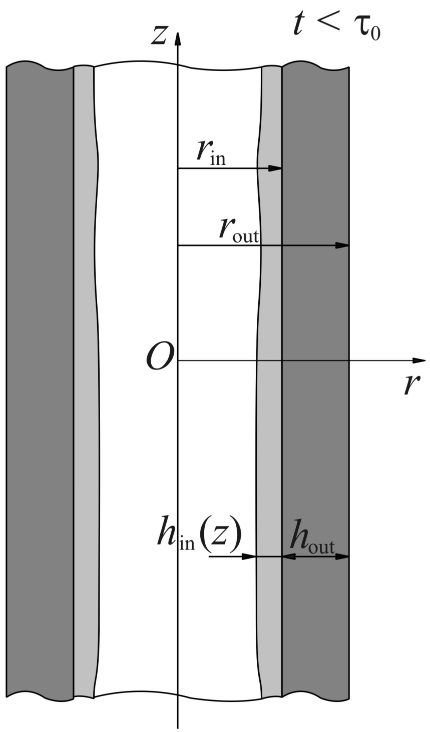
- spraying
- deposition
- immersion in a melt
- enameling
- additive manufacturing
- etc.

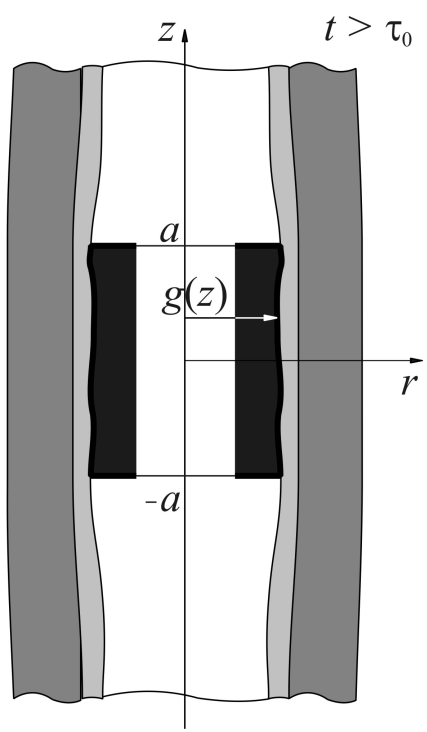






On accounting complex forms of surfaces in the task of interaction of a rigid insert and pipe with internal coating





- Two-layered pipe
- Viscoelastic homogeneous outer layer
- Viscoelastic thin inner layer of **variable thickness**
- Rigid insert with **rough** outer surface
- Known initial tension
- Full contact

What is to be determined?

- Contact stresses in contact region

Mathematical model of problem

Mixed integral equation for finding contact pressure:

$$
\begin{aligned}
&(1-\nu_{\text{in}}^2)h_{\text{in}}(z)\left[\frac{q(z,t)}{E_{\text{in}}(t-\tau_{\text{in}})}-\int_{\tau_0}^{t}\frac{K_{\text{in}}(t,\tau)q(z,\tau)}{E_{\text{in}}(\tau-\tau_{\text{in}})}d\tau\right] \\
&\quad+\frac{2(1-\nu_{\text{out}}^2)}{\pi}\left[\frac{1}{E_{\text{out}}(t-\tau_{\text{out}})}\int_{-a}^{a}k_{\text{c}}\left(\frac{z-\zeta}{r_{\text{in}}}\right)q(\zeta,t)d\zeta\right. \\
&\quad\left.-\int_{\tau_0}^{t}\frac{K_{\text{out}}(t,\tau)}{E_{\text{out}}(\tau-\tau_{\text{out}})}\int_{-a}^{a}k_{\text{c}}\left(\frac{z-\zeta}{r_{\text{in}}}\right)q(\zeta,\tau)d\zeta d\tau\right]=g(z)-[r_{\text{in}}-h_{\text{in}}(z)]
\end{aligned}
$$

$$z\in[-a,a],\quad t\geq\tau_0$$

- Integral operators of different type (mixed integral equation)
- Complex functions in both sizes ($g(z)$, $h_{\text{in}}(z)$)

Mathematical model of problem in dimensionless form

$$c^*(t^*)m^*(z^*)(\mathbf{I}-\mathbf{V}_{\rm in}^*)q^*(z^*,t^*)+(\mathbf{I}-\mathbf{V}_{\rm out}^*)\mathbf{F}^*q^*(z^*,t^*)=-g^*(z^*),\quad z^*\in[-1,1],\quad t^*\geq 1$$

$$\mathbf{V}_{\rm in}^*f(t^*)=\int_1^{t^*}K_{\rm in}^*(t^*,\tau^*)f(\tau^*)d\tau^*,\quad \mathbf{V}_{\rm out}^*f(t^*)=\int_1^{t^*}K_{\rm out}^*(t^*,\tau^*)f(\tau^*)d\tau^*,$$

$$\mathbf{F}^*f(z^*)=\int_{-1}^{1}k^*(z^*,\zeta^*)f(\zeta^*)d\zeta^*$$

Functions connected with complex properties:

$$g^*(z^*)=\frac{r_{\rm in}-h_{\rm in}(z)-g(z)}{a}$$ connect with shape of insert and coating thickness

$$m^*(z^*)=\frac{(1-\nu_{\rm in}^2)h_{\rm in}(z)}{2(1-\nu_{\rm out}^2)a}$$ connect with coating thickness

Mathematical model similar to plane contact problem.

*Manzhirov A V and Kazakov K E 2018 *Math. Models Comput. Simul.* **10** (3) 314–21

Main aspects in approach

1. Using of special representation of unknown function:

$$q^*(z^*,t^*) = \frac{Q(z^*,t^*)}{\sqrt{m^*(z^*)}} - (\mathbf{I} - \mathbf{V}^*_{\text{in}})^{-1} \frac{g^*(z^*)}{c^*(t^*)m^*(z^*)}$$

2. Using a special basis constructed using orthogonalization of a system of functions

$$\left\{ \frac{1}{\sqrt{m^*(z^*)}}, \frac{z^*}{\sqrt{m^*(z^*)}}, \frac{(z^*)^2}{\sqrt{m^*(z^*)}}, \ldots \right\}$$

with subsequent normalization.

Final solution in dimensionless form

$$q^*(z^*,t^*)=\frac{1}{m^*(z^*)}\left[\sum_{k=0}^{\infty} f_k^*(t^*)\varphi_k(z^*)-(\mathbf{I}-\mathbf{V}_{\text{in}}^*)^{-1}\frac{g^*(z^*)}{c^*(t^*)}\right]$$

Functions connected with complex shapes:

$$m^*(z^*)=\frac{(1-\nu_{\text{in}}^2)h_{\text{in}}(z)}{2(1-\nu_{\text{out}}^2)a}\qquad g^*(z^*)=\frac{r_{\text{in}}-h_{\text{in}}(z)-g(z)}{a}$$

Final solution in dimensional form

$$q(z,t)=\frac{a}{h_{\text{in}}(z)}\sum_{k=0}^{\infty}\tilde{f}_k(t)\varphi_k\left(\frac{z}{a}\right)-c_0(t)\frac{r_{\text{in}}-h_{\text{in}}(z)-g(z)}{h_{\text{in}}(z))}$$

Conclusions

- Contact problem for visoelastic tube with inner viscoelastic coating of variable thickness and rigid insert with complex outer shape is posed and solved.
- Analytical representation for contact stresses as a series over special basis is obtained. Complex functions connected with coating properties and insert shape are represented by separate factors and terms.
- It allow one to provide effective numerical calculations using a small number of members of the series. It is important in the case when characteristics of coating and insert described by rapidly changing functions. Another known approaches (using Legendre polynomials, trigonometric functions, etc.) lead us to computational errors.

The work was supported by Ministry of Science and Higher Education within the framework of the Russian State Assignment (contract No. AAAA-A20-120011690132-4) and by the Russian Foundation for Basic Research (project No. 19-51-60001)