Assessment of Possibility of Thermal Fracture in Layered Ceramic Composite

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Problem description

- (1) the heat conduction equation $c\rho \frac{\partial T}{\partial t} = -\frac{1}{2} \left| r k \frac{\partial T}{\partial t} \right| + \frac{U}{2} \left| k \frac{\partial T}{\partial t} \right|$ $\bigg)$ \setminus I \setminus ſ ∂ ∂ ∂ ∂ $|+$ $\bigg)$ \setminus \mathbf{I} \setminus ſ ∂ ∂ ∂ ∂ $=$ ∂ ∂ $\overline{\rho}$ *z T k r z T rk t r r T c* 1
- (2) the equilibrium equations $\frac{CO_{rr}}{2} + \frac{CO_{2r}}{2} + \frac{OF_{rr}}{2} = 0$ σ_{rr} – σ $\ddot{}$ ∂ $\partial \sigma$ $\ddot{}$ ∂ $\frac{\partial \sigma_{rr}}{\partial \theta_{rr}}_{+} \frac{\partial \sigma_{zr}}{\partial \theta_{rr}}_{+} \frac{\sigma_{rr} - \sigma_{\theta\theta_{rr}}}{\partial \theta_{rr}}$ *r z r* $\frac{r}{r} + \frac{v_{0z}}{r} + \frac{v_{rr} - v_{\theta\theta}}{r} = 0$ $\frac{v_{0r}}{r} + \frac{v_{0z}}{r} + \frac{v_{rz}}{r} = 0$ σ $\ddot{}$ ∂ $\partial \sigma$ $\ddot{}$ ∂ $\partial \sigma$ *r z r* $\frac{r_z}{r_z}$ \perp $\frac{C_2}{C_2}$ \perp $\frac{C_{r_z}}{r_z}$
- (3) the strain-displacement relations *dr du^r* ε_{rr} = *dz du^z* ε_{zz} = *r ur* $\varepsilon_{\theta\theta} = \frac{a_r}{r} \varepsilon_{rz} = \frac{1}{2} \left| \frac{m_r}{l} + \frac{m_z}{l} \right|$ \mathbf{r} \setminus $\sqrt{}$ $\mathcal{E}_{rz} = \frac{1}{2} \frac{u u_r}{l} +$ *dz* du_r _{$+$} du_z r_z $\frac{1}{2}$ 1
- (4) the Duhamel–Neumann relations as the constitutive equations

 $\sigma_{ii} = 2\mu\varepsilon_{ii} + \delta_{ii}[\lambda\varepsilon_{kk} - 3K\alpha(T - T_0)]$

- The composite sample under study is a disc 5 mm thick and 30 mm in diameter. The disc has five ceramic layers of the same thickness of 1 mm but of different compositions in accordance with the Figure on this slide. Ceramic layers are connected by sintering at a temperature of 1900 °C, after which they gradually cool down to room temperature 20 °C.
- Due to the cylindrical symmetry of the research object, the problem can be considered in an axisymmetric two-dimensional formulation. The axis of symmetry *z* is directed perpendicular to the plane of the disk, while the axis *r* runs along a radius of the disk. Along the third axis θ, all parameters of the state of stress and strain are considered unchanged due to the homogeneity of the layers. *z*

• Finite element method, ABAQUS/Standard computer code • Axisymmetric continuum finite elements CAX4RT. The uniform mesh with square elements of size 0.05 mm. \Box

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Temperature-dependent MRDMS **material parameters**

- To access the influence of the dependence of material parameters on temperature, we performed different simulations. At first, we adopted that all the physical and mechanical properties do not vary with temperature. Then, we took into account the temperature dependence of thermal and mechanical properties.
- Properties of $ZrO₂$

Equations

 $E(T) = 274.1 - 0.027 T$ [GPa]

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v(T) = 0.3 + 3.2 \cdot 10^{-5} T
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k(T) = 2.072 - 3.656 \cdot 10^{-4} T + 4.347 \cdot 10^{-7} T^2 [W/(m · K)]
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c(T) = 274 + 0.795 T - 6.19 \cdot 10^{-4} T^2 + 1.71 \cdot 10^{-7} T^3 [J/(kg·K)]
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\alpha(T) = 7.091 \cdot 10^{-6} - 2.532 \cdot 10^{-9} T + 2.262 \cdot 10^{-12} T^2 [1/K]
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 $p(T) = 5600/(1+3\alpha(T)(T-293))$ [kg/m³]

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• According to the above mentioned, the system of equations includes

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Numerical method and code

Introduction

Temperature-dependent MRDMS **material parameters**

• Properties of $ZrB_2 - 20\%$ SiC

- In composite materials internal thermal stresses can appear due to the difference in the thermal expansion coefficients of composite constituents. They can play a positive and negative role, either leading to cracking of the material in the case of tensile stresses or restraining the growth of cracks in the case of compressive stress. In recent years, ultra-high-temperature ceramics designed for thermal protection of various products have attracted considerable interest. The presence of several layers in such materials makes it possible to effectively solve not only the problem of thermal protection but also other problems, such as increasing the mechanical strength or chemical resistance. According to the available estimates, the alternation of layers in layered composites leads to the appearance of compressive and tensile internal stresses on different sides of the interface. The choice of the composite constituents and the shape of the products can significantly affect the value of the resulting internal stress. So, in addition to qualitative results on the level and signs of thermal stress, the quantitative assessment of internal thermal stress in products made of composites of a particular composition is of great interest in engineering practice.
- The **aim of this work** is to analyze the internal stresses in a disk-shaped sample of a five-layered ceramic composite consisting of layers of ZrB₂ – 20% SiC with various additives of ZrO₂, raging from 0% to 100 %, when it cools down from the sintering temperature to room temperature.

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 $p(T) = 5520/(1+3\alpha(T)(T-293))$ [kg/m³] 7.8 $\cdot 10^{-6}$ [1/K] if $T \ge 600$ K 5 $\cdot 10^{-6}$ [1/K] if $T < 600$ K $(T) = \begin{cases} 0.18 & \text{if } C \ 7 & \text{if } C \end{cases}$ $c(T) = 622.5 + 0.1T - 1.983 \cdot 10^7 T^{-2}$ [J/(kg·K)] $k(T) = 97.81 - 3.577 \cdot 10^{-2} T + 8.692 \cdot 10^{-6} T^2$ [W/(m · K)] $v(T) = 0.14 + 2.658 \cdot 10^{-6} T + 1.638 \cdot 10^{-5} T^2 - 8.732 \cdot 10^{-12} T^3 + 1.577 \cdot 10^{-15} T^4$ $E(T) = 444.7 - 0.02599 T + 4.541 \cdot 10^{-5} T^2 - 5.213 \cdot 10^{-8} T^3 + 1.261 \cdot 10^{-11} T^4$ [GPa] 6 $\overline{\mathcal{L}}$ $\bigg\}$ $\left\{ \right.$ $\overline{}$ $\cdot 10^{-6}$ [1/K] if $T \geq$ $\cdot 10^{-6}$ [1/K] if T < $\alpha(T) = \begin{cases} 1 & 1 \\ 7 & 8 & 10 \end{cases}$ -*T T T*

Mathematical model

- During cooling of the sintered composite, the fields of strain and stress depend on the temperature field but, in turn, the changes in stress and strain do not affect the temperature, that is, heat is not produced in this case.
- So, to solve the problem, we can perform uncoupled thermal-stress analysis. In this case, a sequential analysis is carried out—first, the formation of the temperature field is calculated, and then the analysis of the stress and strain is performed.
- Since all material parameters (elastic moduli, heat capacity, coefficient of thermal conductivity, linear coefficient of thermal expansion) depend on temperature, the heat transfer problem should be treated as a transient one.
- In this case, modeling can be performed within the framework of quasi-static uncoupled problems of thermoelasticity.
- The problem is considered in an axisymmetric two-dimensional formulation, so we adopt cylindrical coordinate system.
- Hence we get the following set of equations presented in the next slide:

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Boundary conditions

• The initial conditions correspond to the absence of stress and strain at uniform sintering temperature of 1900 °C all over the disk.

- The boundary conditions for the solid mechanics equations correspond to the conditions of axial symmetry at *r* = 0, and free boundary conditions at the outer edge of the disk at *r* = 15 mm and on the planes of the disk at $z = 0$ and $z = 5$ mm.
- For the heat conduction equation, Newton's law of cooling for the heat flux $q = -\beta (T T_{\rm r})$ were set on all faces except *r* = 0 where the symmetry condition was adopted.
- The surface film coefficient β was set as 100 W/(m²·K) during cooling assuming a forced convection in air.

We performed three simulations and compared the results obtained.

- In the first two simulations, we put that all the physical and mechanical properties do not vary with temperature, and equal to the values corresponding to the temperatures of 1900 °C and 20 °C, respectively for the first and second simulations.
- The third simulation took into account the temperature dependence of physical and mechanical properties discussed in the previous section.

• To define the properties of intermediate layers, the rule of mixtures was used.

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Results and discussion

• Distribution of residual stress σ*rr* over the deformed sample. To make a quantitative comparison of the results, we plot the graphs along the three indicated lines: axial line, middle line, and edge line.

Results and discussion

• Plots of residual stress σ_{*rr*} along the axial (center) line. The values are dangerous for possible cracking.

Results and discussion

• Plots of residual stress σ*rr* along the edge line. The values are lower.

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Conclusions

- The temperature dependence of the material parameters makes a great impact on the distribution of residual thermal stress in the ceramic composite, which is obtained by computer simulation.
- If one prefer to take the material parameters as constant for engineering estimation, it is preferable to choose their values for high temperature than for room temperature.
- The level of residual thermal tensile stress is high enough for possible cracking of the ceramic layered composite after cooling.
- The difference in the thermal expansion coefficients of ceramic composite constituents results in buckling of the disk during cooling. To prevent buckling, compressive pressure could be applied to its flat surfaces.